International Journal of Theoretical Physics, Vol. 36, No. 12, 1997

Measurement of Cosmic Background Energy by a Moving Detector in a Riemannian Space-Time

E. Stedile[1,2]

Received May 7, 1997

We investigate the behavior of the thermal energy of a photon gas under the influence of gravity and observed from a moving frame, by considering Boltzmann's equation in a Riemannian manifold. For radiation measurements this approach has a local character, and it points out how the detected energy is affected by the motion of an observer in the presence of gravity.

Lorentz transformations of temperature in special relativity (SR) have long been controversial. According to Einstein (1907), Planck (1908), Tolman (1934), Pauli (1958), and Von Laue (1961), among others, a slow-moving temperature detector endowed with a velocity V should measure an "effective" temperature $T = T_0\sqrt{1 - V^2}$, where T_0 is the temperature of a thermal bath with respect to its own inertial frame (in this paper we adopt natural units, $G = c = k = h = 1$). Later, Ott (1963) and Arzeliès (1965) reached the conclusion that $T = T_0/\sqrt{1 - V^2}$. The question of relativistic temperature transformations was an open problem up to the recent paper of Landsberg and Matsas (1996), who pointed out that there is no Lorentz transformation for temperature.[3] Thus, we can infer that concerning the cosmic microwave background radiation (CMBR), thermal energy is the quantity to be measured in a moving frame, instead of temperature. For the energy spectrum of the CMBR, an important result is given by the Princeton experiment, which leads

[1] Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030.

[2] Permanent address: Department of Physics-UFPR, P.O.B. 19081-81531/990, Curitiba PR, Brazil.

[3] In reality, a directional Lorentz transformation of temperatures given in the form $T(\theta) = \gamma^{-1}T_0(1 - \beta\cos\theta)$ was first proposed in SR by Henry *et al.* (1968). It states that the measured "effective temperature" is not isotropic. However, here we assume a transformation of thermal energy, not temperature, and take gravity into account.

0020-7748/97/1200-2913$12.50/0

to the determination of the Earth velocity with respect to the CMBR (Janssen and Gulkis, 1991; Aldrovandi and Gariel, 1992). Moreover, the measurement of thermal energy on accelerated detectors has emerged as a subject of renewed investigation (Higuchi *et al.*, 1993), and an approach to establish such measurements in a Riemannian space-time, by employing the equivalence principle, was recently proposed by Komar (1995). Here we analyze the detection of the energy spectrum of a photon gas by a moving observer under the influence of gravity. The approach points out a gravitational shift on CMBR signals when measured on an Earth device in comparison with the SR prediction.

We develop a relativistic framework in a Riemannian manifold, taking into account the coordinate time as an evolutionary parameter. For that, we assume the metric tensor in an orthogonal form, where the space-time line element corresponding to a moving particle is

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu = g_{00}\, dt^2 + g_{ij}\, dx^i\, dx^j \tag{1}$$

and whose signature is $(+, -, -, -)$. In this paper Greek indices are space-time indices (0, 1, 2, 3), while Latin indices are space indices (1, 2, 3), and repeated indices are summed over their corresponding ranges. Assuming the coordinate time t as an evolutionary parameter, the coordinate velocity of the particle is $v^i = dx^i/dt$, which yields from (1)

$$ds = (g_{00} + g_{ij} v^i v^j)^{1/2}\, dt \tag{2}$$

Hence, the relativistic action integral

$$A = m \int ds = \int m(g_{00} + g_{ij} v^i v^j)^{1/2}\, dt = \int L\, dt \tag{3}$$

leads to the Lagrangian of the particle

$$L = m(g_{00} + g_{ij} v^i v^j)^{1/2} \tag{4}$$

and to the particle's three-momentum components

$$p_i = \frac{\partial L}{\partial v^i} = \frac{m v_i}{\sqrt{g_{00} - v^2}} \tag{5}$$

where $v^2 = -v_i v^i$,

Under a Legendre transformation of L we obtain for the Hamiltonian

$$H = p_i v^i - L = \frac{g_{00}\, m}{\sqrt{g_{00} - v^2}} \tag{6}$$

and the above results allow us to define a relativistic four-momentum $p_\mu = (H, p_i)$ in such a way that

$$p_\mu p^\mu = m^2 \tag{7}$$

Hence, the Hamiltonian of the particle can be written in terms of p in the form

$$H = \frac{\sqrt{m^2 + p^2}}{\sqrt{g_{00}}} \tag{8}$$

Finally, from (4) and (5) the Lagrange equations of motion yield

$$\frac{dp_i}{dt} = \frac{\partial L}{\partial x^i} = \frac{m}{2\sqrt{g_{00} + g_{ij}v^i v^j}} (\partial_i g_{00} + \partial_i g_{jk} v^j v^k) \tag{9}$$

If we assume an ideal gas as a collection of particles with the same rest mass m, and since the Hamiltonian $\mathcal{H}$ of the gas can be supposed as an additive conserved quantity, then we expect that at thermodynamic equilibrium the mean energy of this gas should be canonically distributed. Thus, for a gas at thermodynamic equilibrium in a Riemannian space-time, we can proceed to model the physics of our system. Hence, the Hamilton equations for the Hamiltonian of the gas are

$$\frac{\partial \mathcal{H}}{\partial p_i} = \frac{dx^i}{dt} = -\frac{g^{ij}p_j}{\sqrt{g^{00}}\sqrt{m^2 + p^2}} \tag{10}$$

$$\frac{\partial \mathcal{H}}{\partial x^i} = -\frac{dp_i}{dt} = -\frac{\mathcal{H}}{2}\frac{\partial(\ln g^{00})}{\partial x^i} - \frac{p_j p_k}{2\sqrt{g^{00}}\sqrt{m^2 + p^2}}\frac{\partial g^{jk}}{\partial x^i} \tag{11}$$

$$\frac{\partial \mathcal{H}}{\partial t} = \frac{d\mathcal{H}}{dt} = \frac{\mathcal{H}}{2}\frac{\partial(\ln g^{00})}{\partial t} - \frac{p_j p_k}{2\sqrt{g^{00}}\sqrt{m^2 + p^2}}\frac{\partial g^{jk}}{\partial t} \tag{12}$$

As usual, the distribution function is $f(\vec{x}_i, \vec{p}_i) = dN/d\Lambda$ in the phase space of the gas, where dN is the number of particles inside the volume element $d\Lambda$. Then, from the left-hand sides of (10) and (11), the total time derivative of $f(\vec{x}_i, \vec{p}_i)$ i.e.,

$$df/dt = (\partial f/\partial x^i)\, dx^i/dt + (\partial f/\partial p^i)\, dp^i/dt + \partial f/\partial t$$

becomes

$$\frac{df}{dt} = \{f, \mathcal{H}\}_{x,p} + \frac{\partial f}{\partial t} \tag{13}$$

where $\{f, \mathcal{H}\}_{x,p}$ is a Poisson bracket.

Let us now analyze the behavior of the thermal energy of a photon gas under the influence of gravity as observed from a moving frame. We recall that an expression for the energy of the CMBR was already derived (see, for instance, Morse, 1965) using the argument of adiabatic expansion of a photon gas. In an alternative way, the behavior of the thermal energy of the CMBR can be derived from the distribution function of the photon gas, with both the effects of the redshift of the spectrum and adiabatic expansion taken into account (Peebles, 1967). It is important to recall that in the standard big-bang model, the thermal spectrum of the CMBR comes from a Planck-type equilibrium before the recombination time, whose form is preserved through the subsequent expansion period and also in a short duration of the recombination era, when photons interact only with the gravitational field. The fact that the spectrum is preserved means that a radiation energy $\overline{\mathscr{E}}$ is fixed by gravity through a time scale parameter $R(t)$. According to Einstein's equation in a matter-dominated homogeneous universe, the product $R(t)\overline{\mathscr{E}}$ is constant, which ensures the preservation of the spectrum. Such a gravitational interaction, although small, is responsible for the equilibrium and it provides for the cosmological redshift. Its existence makes the CMBR to be a "confined" system, with a well-defined "effective" temperature, where gravity plays the role of a "thermal bath" for photons of the CMBR. The interaction energy between photons is supposed strong enough to stablish the equilibrium, but this energy is negligible for the time and dimensions considered in the detection of a sample.

Here we consider the Boltzmann equation in the presence of gravity, by assuming the coordinate time as an evolutionary parameter. Recall that for a photon gas the function $f(\vec{x}_i, \vec{p}_i)$ can be defined at each point in the phase space of the gas, in an element of volume $d\Lambda$, in such a way that there are many photons in that volume element. Moreover, we also assume that the space-time structure of the universe is governed by the Robertson–Walker space-time metric

$$ds^2 = dt^2 - \frac{R^2(t)}{A^2(r)}(dr^2 + r^2\, d\theta^2 + r^2 \sin^2 \theta d\phi^2) \tag{14}$$

where $A^2(r) = 1 + (ar^2/4b^2)$ and $a/b = K$ is a constant denoted the curvature parameter. This yields for the Hamiltonian of the photon gas $\mathscr{H} = p$, according to (8), when extended to a thermodynamic system. The above metric can be written in the form

$$ds = \Gamma^{-1}\, dt \tag{15}$$

where $\Gamma(x^\mu) = (1 - \mathscr{B}^2)^{-1/2}$, with $\mathscr{B} = [R(t)/A(r)]dx^i/dt$, $\mathscr{B}^2 = -\mathscr{B}_i\mathscr{B}^i$, $\mathscr{B}^i = V^i = dx^i/dt$ and $\mathscr{B}_i = -[R^2(t)/A^2(r)]V^i$. If $\mathbf{V}$ denotes the coordinate velocity of an observer with respect to a comoving frame with the photon radiation, then the above result is analogous to the one given by SR. Clearly, absence of gravity means $R(t) = A(r) = 1$, where the form (15) is reduced to

$ds = \gamma^{-1}\,dt$, as predicted by SR, with $\gamma = (1 - V^2)^{-1/2}$ and $V_i = -V^i$. Such analogy yields Lorentz boosts which allow us to perform locally transformations of coordinates under the influence of motion and gravity, by means of the Lorentz boost matrix

$$[L]^{\nu}_{\mu} = \begin{pmatrix} \Gamma & \Gamma\mathcal{B}_1 & \Gamma\mathcal{B}_2 & \Gamma\mathcal{B}_3 \\ \Gamma\mathcal{B}_1 & 1 + \dfrac{(\Gamma - 1)\mathcal{B}_1\mathcal{B}_1}{\mathcal{B}^2} & \dfrac{(\Gamma - 1)\mathcal{B}_1\mathcal{B}_2}{\mathcal{B}^2} & \dfrac{(\Gamma - 1)\mathcal{B}_1\mathcal{B}_3}{\mathcal{B}^2} \\ \Gamma\mathcal{B}_2 & \dfrac{(\Gamma - 1)\mathcal{B}_1\mathcal{B}_2}{\mathcal{B}^2} & 1 + \dfrac{(\Gamma - 1)\mathcal{B}_2\mathcal{B}_2}{\mathcal{B}^2} & \dfrac{(\Gamma - 1)\mathcal{B}_2\mathcal{B}_3}{\mathcal{B}^2} \\ \Gamma\mathcal{B}_3 & \dfrac{(\Gamma - 1)\mathcal{B}_1\mathcal{B}_3}{\mathcal{B}^2} & \dfrac{(\Gamma - 1)\mathcal{B}_2\mathcal{B}_3}{\mathcal{B}^2} & 1 + \dfrac{(\Gamma - 1)\mathcal{B}_3\mathcal{B}_3}{\mathcal{B}^2} \end{pmatrix} \tag{16}$$

and whose determinant is equal to +1, leading thus to proper transformations. In this context, local transformations of the components of a four-vector A^{μ} between two locally inertial frames are given by

$$A'_0 = \Gamma(A_0 - \mathcal{B}\cdot\mathbf{A}), \qquad \mathbf{A}' = \mathbf{A} + \frac{(\Gamma - 1)}{\mathcal{B}^2}(\mathcal{B}\cdot\mathbf{A})\mathcal{B} - \Gamma\mathcal{B}A_0 \tag{17}$$

where the determinant of the Jacobian of such transformations is

$$J = \Gamma\left(1 + \mathcal{B}_1\frac{\partial A^0}{\partial A^1} + \mathcal{B}_2\frac{\partial A^0}{\partial A^2} + \mathcal{B}_3\frac{\partial A^0}{\partial A^3}\right) \tag{18}$$

In particular, if $A_{\mu}A^{\mu}$ = const and if the space components A_i are independent, we obtain that $\partial A^0/\partial A^i = -A_i/A_0$, which allows us to see that $J = A'_0/A_0$, according to the first of Eq. (17). Thus, from the transformation law $dA'_1dA'_2dA'_3 = JdA_1dA_2dA_3$, we conclude that $dA_1dA_2dA_3/A_0 = dA_1dA_2dA_3/A_0$. For the specific case of the relativistic four-momentum, which satisfies the condition (7), we obtain easily that the volume element in the phase space is invariant, under local transformations of coordinates, as pointed out in the literature (see, for instance, Misner, *et al.*, 1973; and Chernikov, 1963). Indeed, we notice that a volume element $d\mathcal{V}$ in the coordinate space and a three-surface element $d\mathcal{P}$ in the momentum space are now transformed respectively by $d\overline{\mathcal{V}} = \Gamma d\mathcal{V}$ and $d\overline{\mathcal{P}} = \Gamma^{-1}\,d\mathcal{P}$, analogously to what happens in SR, and in such a way that the volume element in the phase space $d\Lambda = d\mathcal{V}d\mathcal{P} = d\overline{\mathcal{V}}d\overline{\mathcal{P}}$ remains invariant under the influence of motion and gravity. As a consequence, owing to the definition of the distribution function, this means that $f(\vec{x}_i, \vec{p}_i)$ is also an invariant. This result is a reasonable requirement, because all observers must agree on the number of particles at a given location.

Let us now assume the distribution function of a photon gas, written in terms of the Robertson–Walker metric (14). In this case it is convenient to use the new set of variables (p, q, u) for the three-momentum components of photons, which can be related to the old components (p^r, p^θ, p^ϕ), taking into account that $g_{ij}p^ip^j = -p^2$. These relations are

$$\frac{R}{A} p^r = \mathcal{H}\omega, \qquad \frac{Rr}{A} p^\theta = \mathcal{H} \sin q \sin u, \qquad \frac{Rr \sin \theta}{A} p^\phi = \mathcal{H} \sin q \cos u \tag{19}$$

where $\omega = cos\ q$. In terms of the new variables, the distribution function in a spherically symmetric case turns into $f(r, \mathcal{H}, \omega, t)$, and then its total time derivative becomes

$$df/dt = (\partial f/\partial r)\, dr/dt + (\partial f/\partial p)\, d\mathcal{H}/dt + (\partial f/\partial \omega)\, d\omega/dt + \partial f/\partial t$$

However, from (10), (12), and (19), and considering the metric (14), we obtain for photons

$$\frac{dr}{dt} = -\frac{\omega A}{R}, \qquad \frac{d\mathcal{H}}{dt} = -p\left(\frac{1}{R}\frac{dR}{dt} + \frac{\sin^2 q}{r}\frac{dr}{dt} + \cot\theta \sin^2 q \cos^2 u \frac{d\theta}{dt}\right) \tag{20}$$

We now suppose that the distribution function of the CMBR is isotropic and homogeneous, and that its change due to collisions is negligible. In reality, the mean free path of photons of the CMBR can be assumed as infinite, for the purpose of thermal energy measurements, so there are neither internal interactions nor container border effects to be accounted for. This means that we can consider $\partial f/\partial r = \partial f/\partial w = 0$, which yields $f = f(\mathcal{H}, t)$. Moreover, since the present approach uses the time coordinate to label the variation of a hypersurface in the phase space, then the relativistic Liouville theorem in the presence of gravity is expressed by (Luke and Szamosi, 1970) $df/dt = 0$. With these assumptions, and taking into account that for photons $p = \mathcal{H} = \nu$, we are led to

$$\frac{\partial f}{\partial t} = \nu \frac{d}{dt}(\ln R) \frac{\partial f}{\partial \nu} \tag{21}$$

where ν is the frequency of the CMBR. A general solution to (21) is

$$f(\nu, t) = F[\nu R(t)] \tag{22}$$

where F is an arbitrary function. The conclusion given above is all that we can assert from the Boltzmann equation for a photon gas in the presence of gravity, and observed from a moving frame. However, we recall that classically in any one direction of observation, the CMBR spectrum remains that of a black-body (see, for instance, Bracewell and Conklin, 1968) at the present age t_p of the universe. This result can be obtained if we choose a specific functional form for $F[\nu R(t)]$ in order to agree with observations. Thus, we expect that the distribution function should be at the present time

$$f(\nu, t_p) = \frac{2}{\exp[\nu R(t_p)/\mathscr{E}_p] - 1} \tag{23}$$

where $R(t_p) = 1$, and $\mathscr{E}_p$ is the background thermal energy, which for practical purposes is assumed to correspond to the "effective temperature" $T \approx 2.7$ K. At an arbitrary instant t, the distribution function should be, for the same frequency ν,

$$f(\nu, t) = \frac{2}{\exp[\nu R(t)/\mathscr{E}(t)] - 1} \tag{24}$$

where $\mathscr{E}(t) = R(t)\mathscr{E}_p$ expresses the familiar relation between the thermal energy of the CMBR and the expansion (or contraction) of the universe, as viewed by a moving observer immersed in a gravitational field.

In order to point out the effect of gravity on an energy detector, let us now consider a comoving frame F_0, at rest with respect to the CMBR, and an observer moving with velocity $V << 1$ relative to F_0, and placed at a distance r from the center of a source of gravity whose mass is M. For that we take into account Schwarzschild's metric in spherical coordinates

$$ds^2 = \left(1 - \frac{r_g}{r}\right) dt^2 - \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 - r^2\, d\theta^2 - r^2\sin^2\theta\, d\phi^2 = d\tau^2 - dl^2 \tag{25}$$

where τ is the proper time, $r_g = 2M$ is the gravitational radius of M, and dl is the element of spatial distance along the observer's geodesic line. If we assume the proper velocity of the observer $V = dl/d\tau$, as defined in terms of its proper time $\tau = \sqrt{g_{00}}\, t$ and at a given position r, then (25) becomes

$$ds = \Gamma^{-1}\, dt \tag{26}$$

where we have defined the quantities $\Gamma = (1 - \mathscr{B}^2)^{-1/2}$ and also $\mathscr{B}^2 = V^2 + \delta^2 - V^2\delta^2$, with $\delta = \sqrt{r_g/r}$. This procedure allows us to reduce as before the problem of a moving observer in the presence of gravity to its analog in SR, by replacing γ and V for Γ and $\mathscr{B}$, respectively. It is important to reinforce that in the present case the quantity $\mathscr{B}$ encloses the proper velocity of the observer and the local effect of gravity due to M.

We recall that the CMBR can be assumed as a "confined" system in the sense of Landsberg and Johns (1967). Either in the absence of gravity or if gravity is neglected, our thermodynamic system in its own rest frame F_0 has energy $\mathscr{E}_0$, volume $\mathscr{V}_0$, pressure P_0, and enthalpy $E_0 = \mathscr{E}_0 + P_0\mathscr{V}_0$. However, when observed from a moving frame F which is moving with a "velocity parameter" $\mathscr{B}$ with respect to F_0, it will have an enthalpy $E = \mathscr{E} + P\mathscr{V} = \Gamma E_0 = \sqrt{E_0^2 + p^2}$, where $p = E_0\mathscr{B}/\sqrt{1 - \mathscr{B}^2} = E\mathscr{B}$. Being a confined system, the enthalpy of the CMBR (instead of its energy) constitutes with its three-momentum a four-vector. For an observer with a locally constant "velocity parameter" (i.e., V constant and at a given distance r from the center of M), we can assume that the entropy of the CMBR is the same in both frames F_0 and F, since the distribution function is invariant under the effects of motion and gravity.

Let us now employ the preceding considerations for a background energy detector located on the Earth. First, we recall that the determination of the Earth velocity with respect the CMBR is based on SR arguments, which state that an earth-bound detector measures an angle-dependent thermal energy. However, according to the present approach, the energy as measured in a frame F comoving with the Earth should be (Henry *et al.*, 1968)

$$\mathscr{E}(\theta) = \Gamma^{-1} \frac{\mathscr{E}_0}{1 - \mathscr{B} \cos \theta} = \frac{\mathscr{E}_R}{1 - \mathscr{B} \cos \theta} \tag{27}$$

for a given distance of the Earth from the Sun, where θ is the angle between the normal to the area of the detector and the direction of the Earth's "velocity parameter" $\mathscr{B}$. In the Earth frame, the radiation is specified by the photon distribution in a solid angle $d\Omega$

$$d^3N = \frac{\nu^2\, d\nu\, d\mathscr{V}\, d\Omega}{\exp(\nu/SE) - 1} \tag{28}$$

where d^3N denotes the number of photons detected in F within the volume $d\mathscr{V}$ and at the instant t. The experimental interest lies in the measurement of the intensity of the detected radiation, given in terms of the energy density U. Thus, if we neglect the Sun's radiation and the effects due to the Earth's atmosphere, we get from (27) and (28)

$$\frac{d^2U}{dx\, d\Omega} = \frac{Cx^3}{\exp[x(1 - \mathscr{B} \cos \theta)] - 1} \tag{29}$$

where $x = \nu/\mathscr{E}_R$ and $C = 2(\mathscr{E}_R)^4$. The above result may be written as

$$\frac{d^2U}{dx\, d\Omega} = \frac{Cx^3}{e^x - 1} \left\{1 + \left(\frac{e^x}{e^x - 1}\right)\left[\exp(-\mathscr{B}x \cos \theta) - 1\right]\right\}^{-1} \tag{30}$$

and since for cosmic radiation detected on Earth, $\mathcal{B}x << 1$, we then obtain from (30)

$$\frac{d^2U}{dx\,d\Omega} \approx \frac{Cx^3}{e^x - 1}\left(1 + \frac{\mathcal{B}xe^x\cos\theta}{e^x - 1}\right) \tag{31}$$

which states that the intensity of the detected radiation is a maximum for $\theta = 0$ and a minimum for $\theta = \pi$. If the detected signal is defined by

$$S_{\mathcal{B}} = \left(\frac{d^2U}{dx\,d\Omega}\right)_{\max} - \left(\frac{d^2U}{dx\,d\Omega}\right)_{\min} = \frac{2C\mathcal{B}x^4e^x}{(e^x - 1)^2} \tag{32}$$

and if we assume that the Earth's velocity with respect to the CMBR is (Smoot *et al.*, 1991) $V \approx 0.001237$ and also that the solar gravitational effect on an Earth detector is $\delta \approx 0.000140$, then we conclude that $\mathcal{B} \approx 1.006V$. These values yield from (32) for the detected signal

$$S_{\mathcal{B}} = \frac{2.012CVx^4e^x}{(e^x - 1)^2} \tag{33}$$

This result points out a shift of about 0.6% in the value $S_V = 2CVx^4e^x/(e^x - 1)^2$ predicted by SR (Henry *et al.*, 1968). In Fig. 1 we plot the curves of the detected signals in units of $x = \nu/\mathcal{E}_R$. Notice that the maximum detected signal is near the point $x = 4$ in both cases. It is important to mention that

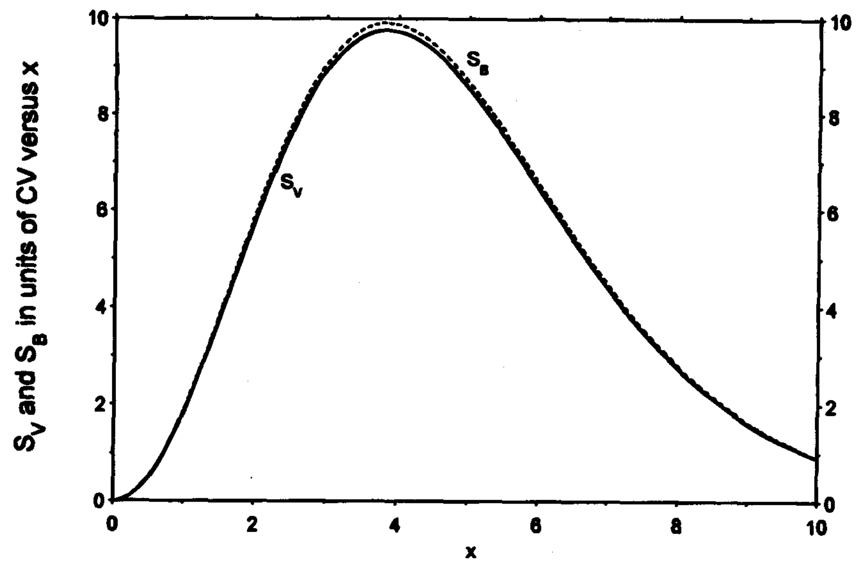

Fig. 1. Detected signals S_B due to motion and gravity (dashed line) and S_V predicted by SR, both in units of CV versus $x = \nu/\mathcal{E}_R$.

in the present framework the effects due to the gravitational shift of frequency of the CMBR at the Earth's position and the nonconstancy of the velocity of light due to solar gravity can both be neglected, since they are lower than the uncertainty on the determination of the Earth's velocity with respect to the CMBR. Moreover, gravitational influences owing to other planets (even at planetary conjunction) and the effect of Earth's gravity can both be neglected. We recall that the velocity V is usually measured by fitting a dipole to the CMBR, with the galactic contribution being carefully subtracted. However, such a contribution to the background radiation cannot be ignored, but can be removed without introducing a systematic bias in a subsequent dipole fit to the CMBR. Although an accurate determination of V is still an experimental challenge, a further statement of $\mathscr{E}(\theta)$ as given by (27) might come from measurements of order V^2, which is not beyond possible improvements of a differential microwave detector.

ACKNOWLEDGMENTS

I am grateful to the Institute for Theoretical Physics, University of California, Santa Barbara, for hospitality, where this paper was concluded, and to G. M. Kremer for a critical reading of this manuscript.

REFERENCES

Aldrovandi, R., and Gariel, J. (1992). *Physics Letters A*, **170**, 5.
Arzeliés, H. (1965). *Nuovo Cimento*, **35**, 792.
Bracewell, R. N., and Conklin, E. K. (1968). *Nature*, **219**, 1343.
Chernikov, N. A. (1963). *Acta Physica Polonica*, **XXXIII**, 5(29), 629.
Einstein, A. (1907). *Jahrbuch Radioaktivita Elektronik*, **4**, 411.
Henry, G. R., Feduniak, R. B., Silver, J. E., and Peterson, M. A. (1968). *Physical Review*, **176**, 1451.
Higuchi, A., Matsas, G. E. A., and Peres, C. B. (1993). *Physical Review D*, **48**, 3731.
Janssen, M. A., and Gulkis, S. (1991). *Proceedings Les Houches Summer School, Design Considerations for the COBE Differential Microwave Radiometers Experiment.*
Komar, A. (1995). *General Relativity and Gravitation*, **27**, 1185.
Landsberg, P. T., and Johns, K. A. (1967). *Nuovo Cimento*, **52**, 28.
Landsberg, P. T., and Matsas, G. E. A. (1996). *Physics Letters A*, **223**, 401.
Luke, S. K., and Szamosi, G. (1970). *Nuovo Cimento*, **LXXB**, 100.
Misner, C. W., Thorne, K. S., and Wheeler, J. A. (1973). *Gravitation*, Freeman, San Francisco.
Morse, P. M. (1965). *Thermal Physics*, Benjamin, New York. p. 351.
Ott, H. (1963). *Zeitschrift für Physik*, **175**, 70.
Pauli, W. (1958). *Theory of Relativity*, Pergamon, Oxford.
Peebles, P. J. E. (1967). Microwave radiation from the big bang, in *Relativity Theory and Astrophysics*, J. Ehlers, ed., Providence, Rhode Island, Vol. 1, p. 274.
Planck, M. (1908). *Annalen der Physik*, **26**, 1.
Smoot, G. F., *et al.* (1991). *Astrophysics Journal*, **371**, L1.
Tolman, R. C. (1934). *Relativity, Thermodynamics and Cosmology*, Clarendon, Oxford.
Von Laue, M. (1961). *Die Relativitatstheorie*, Vieweg, Braunschweig.